

# Goos-Hänchen Shifts of Partially Coherent Light Fields

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We investigate the Goos-Hänchen (GH) shifts of partially coherent fields (PCFs) by using the theory of coherence. We derive a formal expression for the GH shifts of PCFs in terms of Mercer's expansion, and then clearly demonstrate the dependence of the GH shift of each mode of PCFs on spatial coherence and beam width. We discuss the effect of spatial coherence on the resultant GH shifts, especially for the cases near the critical angles, such as totally reflection angle.

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Goos-Hänchen (GH) Shift refers to a tiny (lateral) displacement, from the path expected from geometrical optics, upon total reflection [1]. This effect has been extended into other fields that involve the coherent-wave phenomena, such as neutron waves [2, 3], electron waves [4, 5], and spin waves [6]. It was explained by Artmann [7] that the different transverse wave vectors of a light beam undergo different phase changes and sum of these waves form a reflected beam with a lateral shift. Recently, it was shown [8] that the GH shift is the sum of Renard's conventional energy flux plus a self-interference shift. The self-interference shift originates from the interference between the incident and the reflected beams. Furthermore, it was discovered that the classical Fresnel formulas for laws of refraction and reflection of light are not applicable to partially coherent light [9]. These explanations indicate that the interference or coherence of light is very important to the GH shift.

In 2008, we numerically showed the effect of spatial coherence on the change of the GH shift near the critical angles [10]. Later, an experiment [11] showed the large difference between the measured GH shift of a partially coherent LED light and the theoretical result of a coherent light. However, the very recent investigations [12–16] have raised an important issue “whether the spatial coherence of the partially coherent fields (PCFs) influences the GH shifts?” Although the exact numerical results, calculated from our previous theory [10], are in good agreement with the experimental data [13, 14, 16], it is necessary to reconsider this issue thoroughly and bring to light the role of spatial coherence on the GH shift.

In this Letter, we use the exact theory of coherence to investigate the GH shift of PCFs. First we derive a formal expression to calculate the GH shift of PCFs in terms of the mode expansion of PCFs. Based on this expression, we explain the physical mechanism about the dependence of the GH shift on the spatial coherence and the beam

width. Finally, we suggest a proposal for showing the new effect of the spatial coherence on the practical GH shift below the critical angles.

First we derive the GH shift of PCFs based on the coherence theory [17]. For the two-dimensional PCFs, one usually uses the cross-spectral density (CSD),  $W(x_1, z_1; x_2, z_2, \nu)$ , to describe its propagation, where  $(x_1, z_1)$  and  $(x_2, z_2)$  are the coordinates of the two points in the fields, and  $\nu$  is the frequency of light. For simplicity, we omit the symbol  $\nu$ . According to the theory of coherence,  $W(x_1, z_1; x_2, z_2)$  can be expressed in the form of Mercer's expansion, namely [17]

$$W(x_1, z_1; x_2, z_2) = \sum_m \beta_m \psi_m^*(x_1, z_1) \psi_m(x_2, z_2), \quad (1)$$

where  $\psi_m$  are the eigenfunctions and  $\beta_m \geq 0$  are the corresponding eigenvalues. We rewrite it in another form,

$$W(x_1, z_1; x_2, z_2) = \sum_n \beta_m W^{(m)}(x_1, z_1; x_2, z_2), \quad (2)$$

where  $W^{(m)}(x_1, z_1; x_2, z_2) = \psi_m^*(x_1, z_1) \psi_m(x_2, z_2)$  represents the CSD of a field that is perfect coherent. When PCFs are reflected at the interface ( $z_1 = z_2 = z$ ) between two media, each mode  $\psi_m$  (for both TE and TM polarization) experiences a GH shift. Therefore the reflected CSD for a single mode  $\psi_m$ , at the interface, can be formally written as

$$W_r^{(m)}(x_1, z_1; x_2, z_2) = W_r^{(m)}(x_1, z; x_2, z) = |\bar{r}(\theta_0, \delta\theta_m)|^2 \psi_m^*(x_1 - \Delta_m, z) \psi_m(x_2 - \Delta_m, z), \quad (3)$$

where  $\delta\theta_m$  and  $\Delta_m$  are the angular spread and the practical GH shift of the  $m$ th mode, respectively, and  $\bar{r}(\theta_0, \delta\theta_m)$  is the averaged reflection coefficient within  $\delta\theta_m$  around the incident angle  $\theta_0$  for the  $m$ th mode. Since  $\delta\theta_m$  may be very broad for a large  $m$ , the first-order Taylor expansion (FOTE) on the reflection coefficient  $r$  around  $\theta_0$  can fail [18]. Thus  $\Delta_m$  are very different for

different modes due to the size effect of each mode, and they are also different from the prediction of the formulae  $\Delta_{FOTE} = -\text{Re}[i \frac{\partial \ln r}{\partial \theta} |_{\theta \rightarrow \theta_n}]$  or  $-\frac{\lambda d \phi}{2\pi d \theta}$ , which is based on the stationary phase method under the FOTE [7, 19, 20], here  $\phi_r$  is the phase of  $r$ . Therefore, the total reflected CSD of a PCF at the interface is given by

$$W_r(x_1, z; x_2, z) = \sum_m w_m(\theta_0, \delta\theta_m) \psi_m^*(x_1 - \Delta_m, z) \psi_m(x_2 - \Delta_m, z), \quad (4)$$

where  $w_m(\theta_0, \delta\theta_m) = |\bar{r}(\theta_m, \delta\theta_m)|^2 \beta_m$  represents the weight of the  $m$ th reflected mode. Then the intensity of the reflected beam is

$$I_r(x, z) = \sum_m w_m(\theta_0, \delta\theta_m) |\psi_m(x - \Delta_m, z)|^2. \quad (5)$$

Using the normalized first moment of the light field [21, 22],  $\Delta = \int x I_r(x, z) dx / \int I_r(x, z) dx$ , we obtain the resultant GH shift as follows

$$\Delta = \frac{\sum_m w_m(\theta_0, \delta\theta_m) \Delta_m}{\sum_m w_m(\theta_0, \delta\theta_m)}, \quad (6)$$

where the normalization condition,  $\int |\psi_n(x, z; \nu)|^2 dx = 1$ , has been used. Equation (6) is a formal expression for calculating the practical GH shift of PCFs. This result is different from that in Refs. [12] and [13]. In Refs. [12] and [13], since all shifts  $\Delta_m$  are assumed to be  $\Delta_{FOTE}$ , so that  $\Delta = \Delta_{FOTE}$  is independent of spatial coherence. However, this is not true for PCFs, especially for the incoherent light fields. In the following discussion, we will see that, as  $m$  increases, there is a large difference between  $\Delta_m$  and  $\Delta_{FOTE}$ . Even for a coherent beam,  $\Delta_m$  also changes due to the finite-size effect of practical light beams [23, 24]. Thus the exact expression for  $\Delta_m$  for each mode should be defined as [21, 22]

$$\Delta_m = \int x |\psi_m^r(x, z)|^2 dx / \int |\psi_m^r(x, z)|^2 dx, \quad (7)$$

where  $\psi_m^r$  is the reflected field of the  $m$ th mode at the interface. Therefore, for an incoherent light field, we must include the contributions of the shifts  $\Delta_m$  of all modes to the resultant GH shift  $\Delta$ .

Next we consider how/why the spatial coherence affects the GH shift of each mode of PCFs. We briefly review a famous example: Gaussian Shell-model (GSM) beam, which is an excellent model for describing PCFs [17]. The normalized eigenfunctions and eigenvalues of GSM beams are given by [17] (also see Refs. [25, 26])

$$\psi_m(x) = \left(\frac{2c}{\pi}\right)^{1/4} \frac{1}{(2^m m!)^{1/2}} H_m[x(2c)^{1/2}] e^{-cx^2}, \quad (8)$$

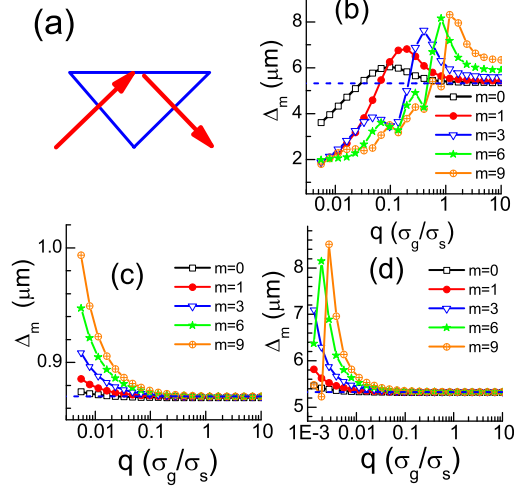


FIG. 1: (Color) (a) The schematic of total reflection from a prism. (b-d) The dependence of the GH shifts  $\Delta_m$  of each mode on the spatial coherence ( $q$ ) at different values of  $\theta_0$ : (b,d)  $\theta_0 = 41.5^\circ$  and (c)  $\theta_0 = 45^\circ$ . In (b and c)  $\sigma_s = 0.1 \text{ mm}$ , and in (d)  $\sigma_s = 2 \text{ mm}$ . The blue dashed lines in (b, c, and d) denote the values of  $\Delta_{FOTE}$ .

and  $\beta_m = A^2 \left(\frac{\pi}{a+b+c}\right)^{1/2} \left(\frac{b}{a+b+c}\right)^m$ , where  $H_m(x)$  are the Hermite polynomials,  $a, b$  and  $c$  are positive quantities and are defined as:  $a = (4\sigma_s^2)^{-1}$ ,  $b = (2\sigma_g^2)^{-1}$ ,  $c = [a^2 + 2ab]^{1/2}$ . Here  $\sigma_s$  and  $\sigma_g$  are the beam half-width and the spectral coherence width of PCFs, respectively. The ratio of the eigenvalue  $\beta_m$  to the lowest eigenvalue  $\beta_0$  is evidently given by [17]

$$\frac{\beta_m}{\beta_0} = \left[ \frac{1}{(q^2/2) + 1 + q[(q/2)^2 + 1]^{1/2}} \right]^m, \quad (9)$$

where  $q = \sigma_g/\sigma_s$  is a measure of the degree of global coherence of a GSM source. Obviously, for  $q \gg 1$  ( $\sigma_g \gg \sigma_s$ ),  $\beta_m/\beta_0 \approx q^{-2m}$ . This implies that, for all  $m > 0$ ,  $\beta_m \ll \beta_0$  and hence the behavior of the beam is well approximated by the lowest-order mode. However, for  $q \ll 1$  ( $\sigma_g \ll \sigma_s$ ),  $\beta_m/\beta_0 \approx 1 - mq$ . Thus, for a very incoherent light, a large number of modes (of the order  $1/q$ ) are needed to represent the light field adequately.

Since each mode of PCFs is perfectly coherent, it is easy to obtain the GH shift for each mode under a certain incident angle upon total internal reflection, as illustrated in Fig. 1(a). Here we use the coherent angular-spectral theory [9, 19, 22, 27]. From Eq. (8), we readily obtain its angular spectrum,  $\tilde{\psi}_m(k_x)$ , by using a Fourier transformation. For an inclined case with  $\theta_0 > 0$ ,  $\tilde{\psi}_m(k_x)$  becomes  $\tilde{\psi}_m(k_x - k_{x0})$  with the replacement  $\sigma_s \rightarrow \sigma_s \sec \theta_0$  and  $\sigma_g \rightarrow \sigma_g \sec \theta_0$ , where  $k_x$  is the transverse component of the wavenumber  $k$  of light in the first medium, and  $k_{x0} = k \sin \theta_0$ . Therefore the reflected field of the

$m$ th mode is given by

$$\psi_m^r(x) = \frac{1}{\sqrt{2\pi}} \int r(k_x) \tilde{\psi}_m(k_x - k_{x0}) \exp[ik_x x] dk_x. \quad (10)$$

Then, using Eq. (7), we can obtain all shifts  $\Delta_m$  in any situation. In the following calculations, we take the refractive index of the prism  $n = 1.514$  at wavelength  $\lambda = 675$  nm, so the critical angle of the totally internal reflection is  $\theta_c = 41.34^\circ$ . Here we only present the result for the TM polarization, due to the similarity between TM and TE polarizations.

*Effect of spatial coherence.*—Figures 1(b) and 1(c) show the typical dependence of the GH shifts  $\Delta_m$  of the  $m$ th mode on the spatial coherence ( $q$ ) under different values  $\theta_0$ : (b)  $\theta_0 = 41.5^\circ$  and (c)  $\theta_0 = 45^\circ$ . In these two cases, we take  $\sigma_s = 0.1$  mm ( $\gg \lambda$ ). From Figs. 1(b) and 1(c), it is found that, near the critical angle  $\theta_c$ , the absolute shifts  $\Delta_m$  are strongly dependent on  $q$ . For  $m = 0$ , the value  $\Delta_0$  slightly increases when  $q$  is gradually close to 0.1, and then it decreases as  $q$  further decreases. As  $m$  increases, the changes  $\Delta_m$  become more dramatic with the decreasing  $q$ , and more oscillations appear due to the fact that the part components of  $\tilde{\psi}_m(k_x - k_{x0})$  have been cut off below  $\theta_c$  as  $\tilde{\psi}_m(k_x - k_{x0})$  is broadened with the decreasing of  $q$ . From Fig. 1(c), for the cases of  $\theta_0$  being far away from  $\theta_c$ , the values  $\Delta_m$  change much more for larger  $m$ . Thus it is expected that there must be a difference between the coherent and incoherent limits [10, 15, 16]. Comparing Fig. 1(b) with Fig. 1(c), it is also found that the changes of  $\Delta_m$  near  $\theta_c$  are more remarkable than that for the cases being away from  $\theta_c$ .

In Fig. 1(d), we plot another situation for the dependence of  $\Delta_m$  on  $q$  at  $\theta_0 = 41.5^\circ$ , with  $\sigma_s = 2$  mm. Although  $\theta_0$  is still near to  $\theta_c$ , the changes of  $\Delta_m$  in Fig. 1(d) are considerably small for  $q \geq 0.01$ . This is due to the effect of beam width ( $2\sigma_s$ ) on  $\Delta_m$  discussed below. From Fig. 1(d), it is clear that there is a large difference between  $\Delta_m$  and  $\Delta_{FOTE}$  in the incoherent limit ( $q < 0.01$ ). When  $m$  increases, some oscillations may also appear for a sufficient small  $q$ .

In Fig. 2, we further show the changes of  $\Delta_m$  as a function of  $m$  under two limits: (1)  $q = 10$  and (2)  $q = 0.01$  with (a, c)  $\theta_0 = 41.5^\circ$  and (b, d)  $\theta_0 = 45^\circ$ . Insets in Figs. 2 (a, c) show the value of  $\beta_m/\beta_0$  as a function of  $m$  for  $q = 10$  and  $q = 0.01$ , respectively. For the fully coherent limit ( $q \gg 1$ ), when  $\theta_0$  is close to  $\theta_c$  [see Fig. 2(a)],  $\Delta_m$  vary dramatically as  $m$  increases; while when  $\theta_0$  is far away from  $\theta_c$  [see Fig. 2(b)],  $\Delta_m$  are nearly independent of  $m$  and they are overlapped with the corresponding value  $\Delta_{FOTE}$ . Thus, in the full-coherent limit,  $\Delta_m$  are independent of  $m$  only under the cases of  $\theta_0$  being far away from  $\theta_c$ . Meanwhile it is only the shifts  $\Delta_0$  of the lowest mode ( $m = 0$ ) that mainly contribute to the resultant GH shift  $\Delta$  since  $\beta_m$  do decrease quickly for  $m > 0$ , see the inset in Fig. 2(a). For the completely incoherent limit ( $q \ll 1$ ), see Figs. 2(c, d), whether  $\theta_0$  is close to

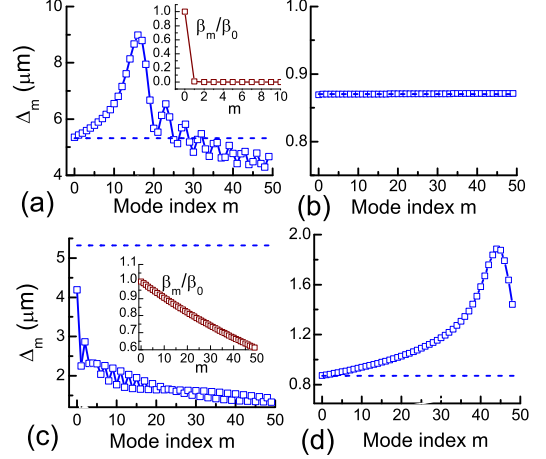


FIG. 2: (Color) The dependence of the GH shifts  $\Delta_m$  of each mode on the mode index  $m$  under two limits (a, b)  $q = 10$  and (c, d)  $q = 0.01$  with incident angles (a, c)  $\theta_0 = 41.5^\circ$  and (b, d)  $\theta_0 = 45^\circ$ . Insets in (a) and (c) show the value of  $\beta_m/\beta_0$  as a function of  $m$  for  $q = 10$  and  $q = 0.01$ , respectively. The dashed lines in (a-d) denote the values of  $\Delta_{FOTE}$ .

or far away from  $\theta_c$ ,  $\Delta_m$  do vary as  $m$  increases; and the contributions of the higher-order modes must be included since  $\beta_m$  changes very slowly for  $m > 0$ , see the inset in Fig. 2(c). This leads to the resultant GH shift  $\Delta$  deviated from the full-coherent limit.

*Effect of beam width ( $2\sigma_s$ ).*— We note that the beam width of the PCFs plays a role on the GH shift, since the effective width ( $2\sigma_m^{eff}$ ) of each mode  $\psi_m$  is related to both  $\sigma_s$  and  $\sigma_g$  [17]. From Eq. (8), we can obtain  $\sigma_m^{eff} = \sqrt{2m+1}\sigma_s/[1+(4/q^2)]^{1/4}$  and its corresponding angular spread  $\delta\theta_m = \frac{180\sqrt{2m+1}}{\pi k\sigma_s}[1+(4/q^2)]^{1/4}$  (in the unit of degree). In order to keep the fixed values,  $\sigma_m^{eff}$  and  $\delta\theta_m$ , if  $\sigma_s$  increases, the value of  $q$  must decrease. In other words, for a fixed value of  $q$ , if  $\sigma_s$  increases, then  $\sigma_m^{eff}$  increases but  $\delta\theta_m$  decreases. This means that increasing  $\sigma_s$  suppresses the effect of spatial coherence ( $q$ ) on the GH shift. By comparing Fig. 1(b) and Fig. 1(d), it is clear that, increasing  $\sigma_s$  leads to the weakening of the effect of spatial coherence on the GH shift. It should be pointed out that, for a coherent beam, the effect of beam width has been investigated in the very early literature [18, 23] and has also been experimental demonstrated [21, 28]. Therefore it is expected that the beam width has also an effect on the GH shift for PCFs.

Figure 3 shows the detailed effect of  $\sigma_s$  on the GH shift  $\Delta_m$  near  $\theta_c$ . From Fig. 3(a), even for a full-coherent limit with  $q = 10$ , when  $\sigma_s$  is small enough ( $< 0.3$  mm), the values  $\Delta_m$  begin to be significantly different from the value of  $\Delta_{FOTE}$ , and such a difference becomes larger as  $m$  increases. Remember that it is only the lowest

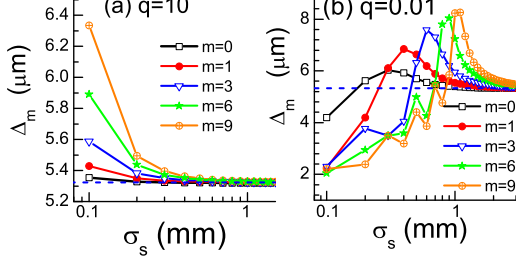


FIG. 3: (Color) Effect of beam half-width on the GH shifts  $\Delta_m$  of each mode with different fixed  $q$ : (a)  $q = 10$ , and (b)  $q = 0.01$ , with  $\theta_0 = 41.5^\circ$ .

mode ( $m = 0$ ) that dominates the resultant GH shift  $\Delta$  in the full-coherent limit, thus other  $\Delta_m$  with  $m > 0$  have no contributions to  $\Delta$ . However, in the incoherent limit ( $q = 0.01$ ), see Figs. 3(b), when  $\sigma_s$  is larger than 2 mm, the difference between  $\Delta_m$  and  $\Delta_{FOTE}$  gradually disappears due to the suppressing effect of  $\sigma_s$  on  $\Delta_m$ ; while for the cases when  $\sigma_s$  is smaller than 2 mm in our cases,  $\Delta_m$  change dramatically and they are very different from  $\Delta_{FOTE}$ .

In fact, on comparing Fig. 3(b) with Fig. 1(b), we find that the role of  $\sigma_s$  on  $\Delta_m$  for a small  $q$  is similar to the role of  $q$  on  $\Delta_m$  for a small  $\sigma_s$ . On comparing Fig. 3(a) with Fig. 1(d), we can also find that the role of  $\sigma_s$  on  $\Delta_m$  for a large  $q$  is similar to the role of  $q$  on  $\Delta_m$  for a large  $\sigma_s$ . Therefore, both  $\sigma_s$  and  $q$  have the equivalent role on the GH shift.

Now we have known the roles of  $\sigma_s$  and  $q$  on the GH shift of each mode of PCFs, and have explained why/how they affect the shift  $\Delta$ . However, it is inconvenient for using Eq. (6) to obtain  $\Delta$  since it is time-consuming to know all the practical shifts  $\Delta_m$  for PCFs when  $q$  is very small. For example, if  $q = 0.01$ , we need 100. modes at least. There is a much realistic method to directly obtain  $\Delta$ . Based on our previous investigation, the exact expression for the intensity of the reflected PCFs, at the interface of two media ( $z_1 = z_2 = 0$ ), can be given by [10],

$$\begin{aligned} I_r(x, 0) &= W_r(x, 0; x, 0) \\ &= \frac{1}{2\pi} \int \int r^*(k_{x1}) r(k_{x2}) W_i(k_{x1}, 0; k_{x2}, 0) \\ &\quad \times \exp[-i(k_{x1} - k_{x2})x] dk_{x1} dk_{x2}, \end{aligned} \quad (11)$$

where  $W_i(k_{x1}, 0; k_{x2}, 0)$  is the initial CSD in the spatial angular-frequency domain at  $z = 0$ , and  $r(k_{x2})$  is the reflection coefficient. Substituting Eq. (11) into the definition of  $\Delta$ , we can obtain the GH shift of PCFs at  $z = 0$  by the exact numerical method.

Finally, let us briefly discuss how to experimentally demonstrate the effect of spatial coherence on the GH

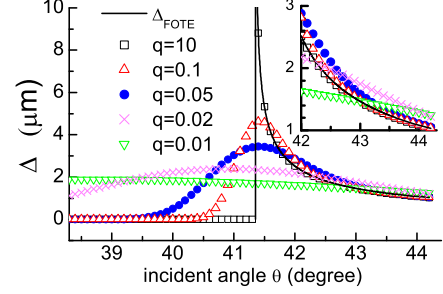


FIG. 4: (Color) The resultant GH shifts as a function of incident angle under different values of  $q$  with a fixed value of  $\sigma_s = 0.2$  mm. Insets are the details above  $\theta_c$ .

shift of PCFs, since the recent experiments [13, 14] have not revealed this effect. From the above discussion, we have already seen that the large value  $\sigma_s$  weakens the effect; and near  $\theta_c$ , the spatial coherence has a larger effect. Thus one should take a small  $\sigma_s$  and measure the absolute GH shift  $\Delta$  of PCFs near  $\theta_c$  in the experiment. In Fig. 4, we predict a dependence of its spatial coherence on the absolute GH shifts for experimental reference. In this case, we take  $\sigma_s = 0.2$  mm, and consider five cases:  $q = 10, 0.1, 0.05, 0.02$  and  $0.01$ . From Fig. 4, we see that, for a full coherent light, there are non-zero GH shifts above  $\theta_c$  but zero below  $\theta_c$ , and all the shifts for  $\Delta$  are overlapped with the curves of  $\Delta_{FOTE}$ . However, for a PCF or an incoherent light field, the GH shifts above  $\theta_c$  may be smaller or larger than  $\Delta_{FOTE}$ , see the insets in Fig. 4. More importantly, the GH shifts below  $\theta_c$  are no longer equal to zero. This is a distinct result for PCFs, which is completely different from the full-coherent prediction. Actually, the latter effect has been observed in a recent experiment [11], where the authors observed a non-zero GH shift below the critical angle, but they cannot explain it. The non-zero GH shifts of PCFs below  $\theta_c$  are very similar to the effect of the narrow beam width of the coherent beam on the GH shifts [23, 29]. Since our curves in Fig. 4 have the same property with other experiments [21, 30, 31], we hope our suggestions could lead to a direct experimental observation of this effect in the system of the total internal reflection.

In summary, we have found that both the spatial coherence and beam width of PCFs have strong effect on its GH shift, which are explained by the formal equation (6) by using the exact theory of coherence. Our results show that the spatial coherence of PCFs play an important role to determine the resultant GH shifts. Finally, we suggest a potential experiment to demonstrate this effect and display a distinct effect for experimental verification. These effects are very important to the applications of the GH shift in nano- or micro-scaled structures [11, 30], where



light beams are usually focused into the small region and the coherence may play an important role. Our results are also important to the applications of the GH effect in other fields, such as neutron systems [2, 3] and electronic systems [4, 5], where the coherent sources are usually not available.

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